Unitarization in Kaluza-Klein theory and the Geometric Bootstrap

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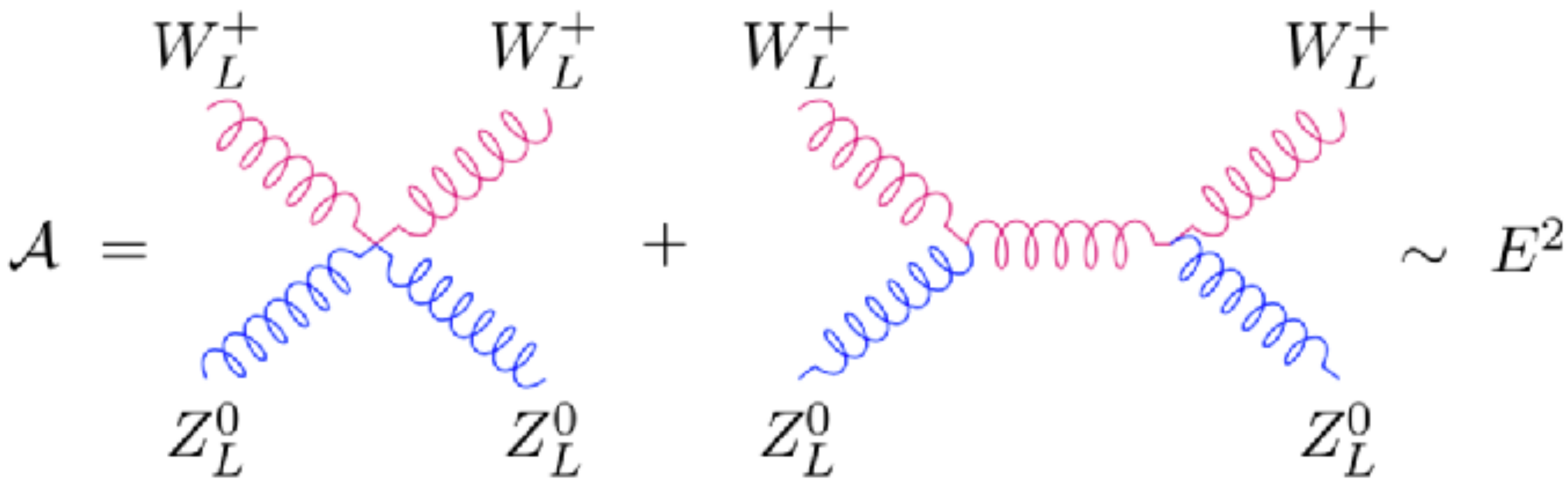
ITMP, Moscow State University, Oct. 13, 2021

James Bonifacio, KH: arxiv:1910.04767, arxiv:2007.10337

James Bonifacio: arxiv:2107.09674

Massive spin-1 scattering in the Standard Model

Scattering of longitudinal modes of W, Z bosons:

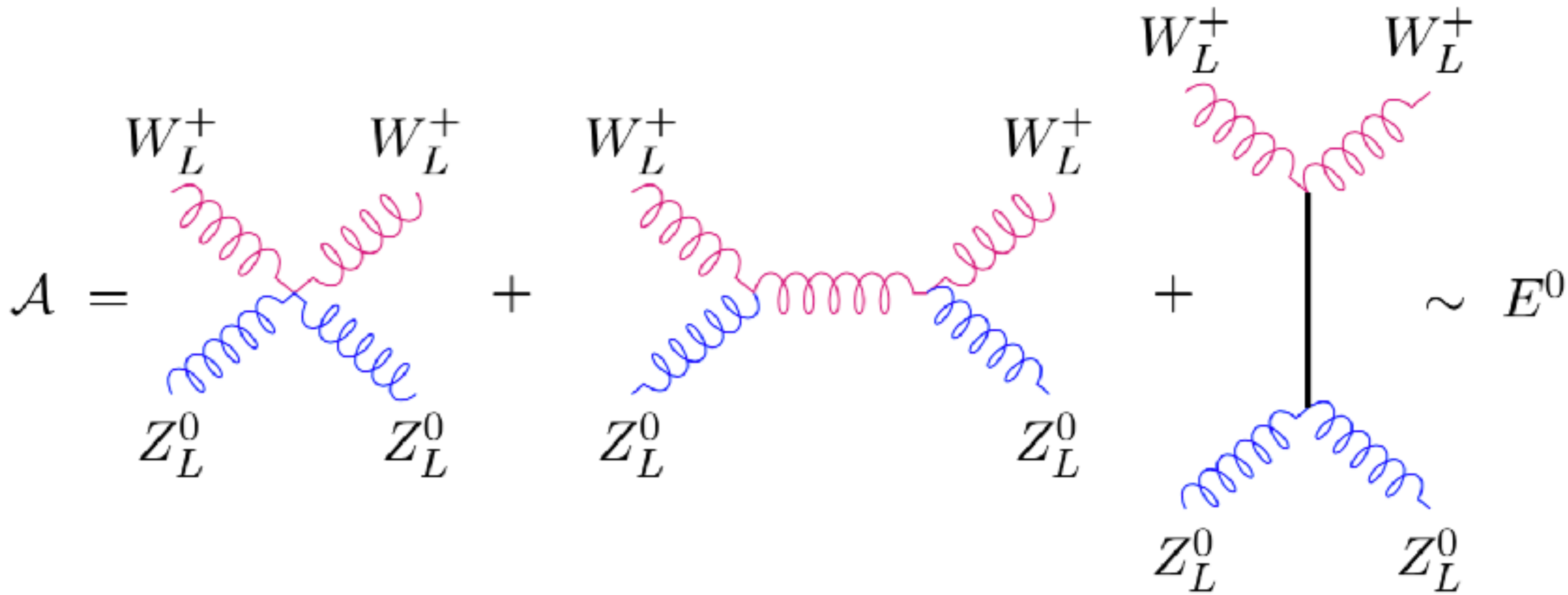


Grows with energy, violates perturbative unitarity at ~ 1 TeV

Something interesting must happen before this scale: no-lose theorem for LHC

Higgs mechanism

Adding a scalar softens high-energy behavior:



Restores perturbative unitarity

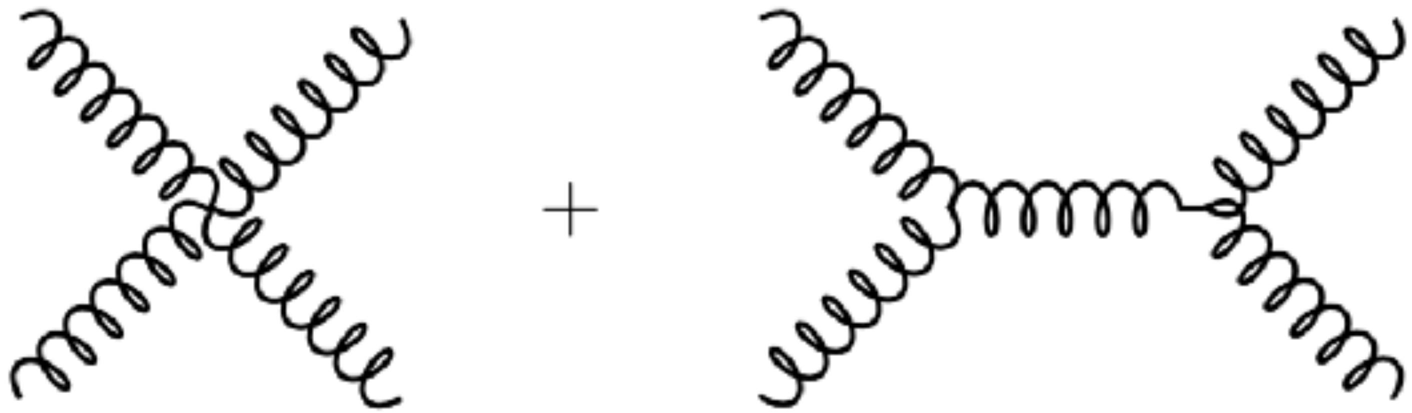
Weakly coupled UV completion (Higgs mechanism)

Massive spin-2 scattering

Generic interactions (Einstein-Hilbert plus a graviton potential)

Arkani-Hamed, Georgi, Schwartz (2003)

$\mathcal{A} =$



$\sim E^{10}$

For special choices of interaction (dRGT massive gravity), this can be improved to E^6

de Rham, Gabadadze, Tolley (2010)

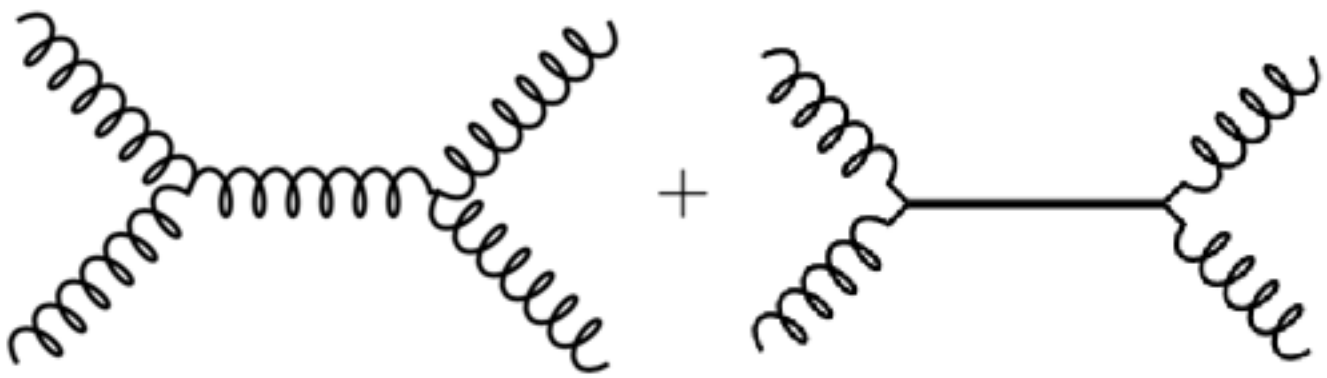
This is the best that can be done without new particles

James Bonifacio, KH (1804.08686)

Gravitational Higgs mechanism?

Can we do better by adding a finite number of new particles with spin < 2?

$\mathcal{A} =$



$+ \cdots \sim E^{p<6}$

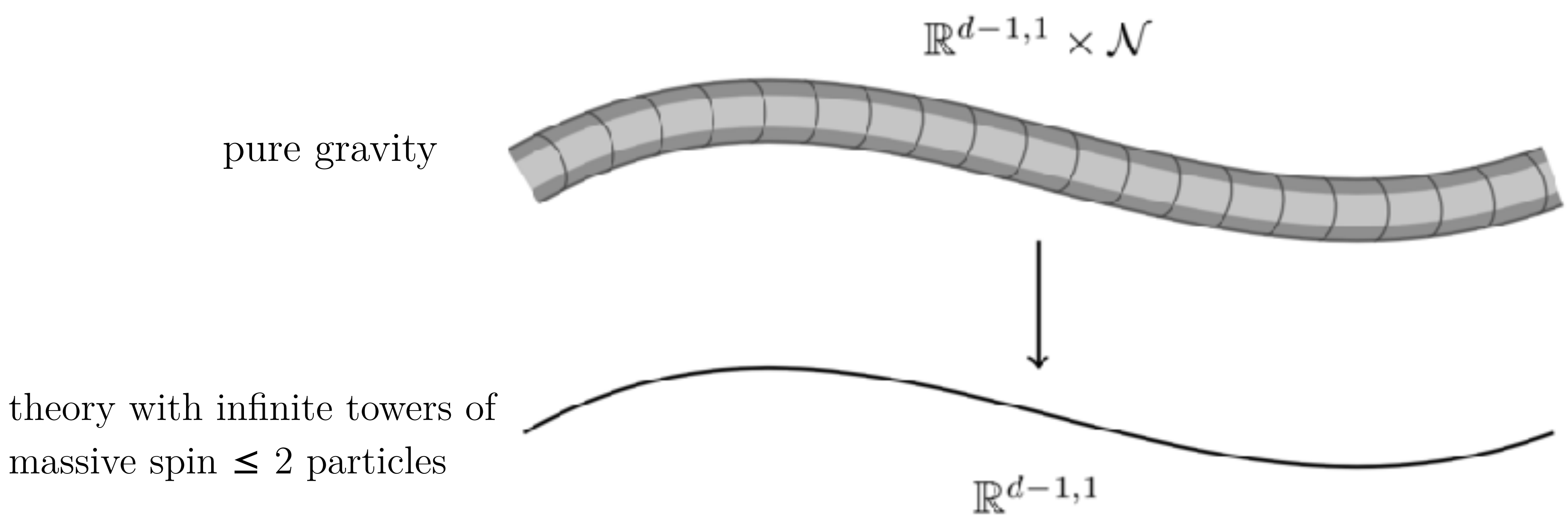
No.

James Bonifacio, KH, Rachel Rosen (1903.09643)

Kaluza-Klein theory

We know that we should be able to do better by adding an infinite number of new particles with spin ≤ 2

$$ds^2 = \bar{G}_{A_1 A_2} dX^{A_1} dX^{A_2} = \eta_{\mu\nu} dx^\mu dx^\nu + \underbrace{\gamma_{mn} dy^m dy^n}_{N \text{ compact smooth dimensions}}$$



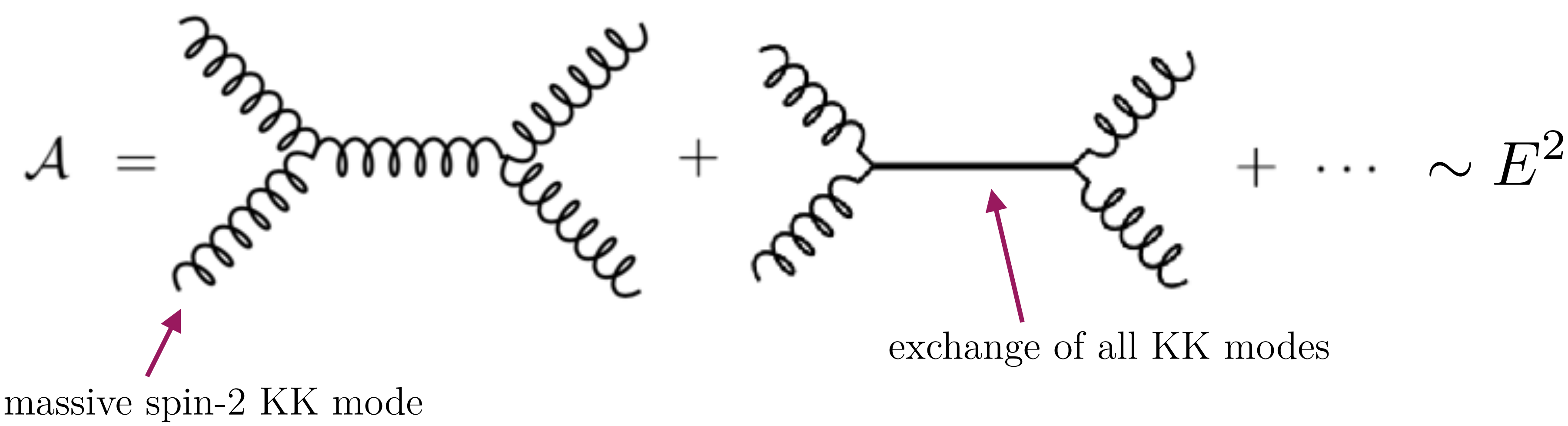
Kaluza-Klein amplitudes

James Bonifacio, KH (1910.04767)
Chivukula, Foren, Mohan, Sengupta, Simmons (2019-2020)

Higher dimensional theory is pure GR ➡ Graviton amplitudes grow with energy like $\sim E^2$

Lower dimensional theory, keeping all KK modes, is just a re-writing of higher dimensional GR ➡ amplitudes should still grow like $\sim E^2$

Lower dimensional theory has massive spin-2 states in the spectrum: how is their high energy scattering softened to E^2 ?



Kaluza-Klein theory

$$ds^2 = \bar{G}_{AB} dX^A dX^B = \eta_{\mu\nu} dx^\mu dx^\nu + \underbrace{\gamma_{mn} dy^m dy^n}_{N \text{ compact smooth dimensions}}$$

Higher dimensional Einstein equations

↓

Internal manifold is an *Einstein manifold* : $$R_{mn}(\gamma) = \lambda\, \gamma_{mn}$$

Non-trivial constraints will require this condition

Lower dimensional spectrum is determined by various Laplacians on the internal manifold

- scalar (ordinary Laplacian)
- vector (Hodge Laplacian)
- tensor (Lichnerowicz Laplacian)

Scalar Laplacian

$$\Delta\psi_a \equiv -\Box\psi_a = \lambda_a\psi_a, \qquad \lambda_a > 0$$

Orthonormality

$$\int_{\mathcal{N}} \psi_{a_1}\psi_{a_2} = \delta_{a_1 a_2},$$

Completeness

$$\phi = \frac{c^0}{V^{1/2}} + \sum_a c^a \psi_a,$$

zero mode (constant)

Conformal scalars (exist only on round spheres)

$$\left(\nabla_m\nabla_n - \frac{1}{N}\gamma_{mn}\Box\right)\psi_a = 0, \quad a \in I_{\text{conf.}},$$

Lichnerowicz bound

$$\lambda_a \geq \frac{R}{N-1}$$

saturated only by conformal scalars

Vector Laplacian

Hodge Laplacian:

$$\Delta Y_{m,i} \equiv -\Box Y_{m,i} + R_m{}^n Y_{n,i} = \lambda_i Y_{m,i}, \quad \nabla^m Y_{m,i} = 0, \qquad \lambda_i \geq 0$$

Orthonormality

$$\int_{\mathcal{N}} Y_{m,i_1} Y^m_{i_2} = \delta_{i_1 i_2}.$$

Completeness (Hodge decomposition)

$$V_m = \sum_i c^i Y_{m,i} + \sum_a c^a \partial_m \psi_a,$$

Killing vectors

$$\nabla_{(m} Y_{n),i} = 0, \quad i \in I_{\text{Killing}}.$$

Bound

$$\lambda_i \geq \frac{2R}{N}$$

↑ saturated only by Killing vectors

Tensor Laplacian

Lichnerowicz Laplacian:

$$\Delta_L h^{TT}_{mn,\mathcal{I}} \equiv -\Box h^{TT}_{mn,\mathcal{I}} + \frac{2R}{N} h^{TT}_{mn,\mathcal{I}} - 2R_m{}^p{}_n{}^q h^{TT}_{pq\mathcal{I}} = \lambda_{\mathcal{I}} h^{TT}_{mn,\mathcal{I}}, \qquad \nabla^m h^{TT}_{mn,\mathcal{I}} = h^{TT\,m}_m = 0,$$

Orthonormality

$$\int_{\mathcal{N}} h^{TT}_{mn,\mathcal{I}_1} h^{mn,TT}_{\mathcal{I}_2} = \delta_{\mathcal{I}_1 \mathcal{I}_2}.$$

Completeness (symmetric tensor Hodge decomposition)

$$T_{mn} = \sum_{\mathcal{I}} c^{\mathcal{I}} h^{TT}_{mn,\mathcal{I}} + 2 \sum_{i \notin I_{\text{Killing}}} c^i \nabla_{(m} Y_{n),i} + \sum_{a \notin I_{\text{conf.}}} \tilde{c}^a \left(\nabla_m \nabla_n \psi_a - \frac{1}{N} \nabla^2 \psi_a \gamma_{mn} \right)$$
$$+ \sum_a \frac{1}{N} c^a \psi_a \gamma_{mn} + \frac{1}{N V^{1/2}} c^0 \gamma_{mn},$$

moduli space of Einstein structures (“zero” modes)

$$\lambda_{\mathcal{I}} \; = \; 2R/N$$

No known general lower bound (there may be finite number of negative eigenvalues)

Spectrum

KH, Janna Levin, Claire Zukowski (1310.6353)

Expand metric over eigenfunctions:

$$G_{AB} = \bar{G}_{AB} + H_{AB} \quad , \qquad H_{A_1 A_2} = \begin{pmatrix} H_{\mu\nu} & H_{\mu n} \\ H_{m\nu} & H_{mn} \end{pmatrix} \quad ,$$

$$H_{\mu\nu}(x,y) = \sum_a h^a_{\mu\nu}(x)\psi_a(y) + \frac{1}{\sqrt{V}} h^0_{\mu\nu}(x),$$

$$H_{\mu n}(x,y) = \sum_i A^i_\mu(x) Y_{ni}(y) + \sum_a A^a_\mu(x)\partial_n \psi_a(y),$$

$$H_{mn}(x,y) = \sum_{\mathcal{I}} \phi^{\mathcal{I}}(x) h^{TT}_{mn,\mathcal{I}}(y) + \sum_{i \notin I_{\text{Killing}}} \phi^i(x) \nabla_{(m} Y_{n)i}(y)$$

$$+ \sum_{a \notin I_{\text{conf.}}} \tilde{\phi}^a(x) \left(\nabla_m \nabla_n \psi_a(y) - \frac{1}{N} \nabla^2 \psi_a(y) \gamma_{mn} \right)$$

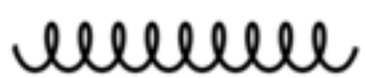
$$+ \frac{\gamma_{mn}}{N} \left[\sum_a \phi^a(x) \psi_a(y) + \frac{1}{\sqrt{V}} \phi^0(x) \right] .$$

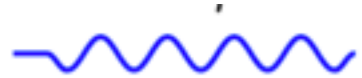
Spectrum

KH, Janna Levin, Claire Zukowski (1310.6353)

Lower dimensional spectrum:

massless graviton: $h^0_{\mu\nu}$ 

massive gravitons: $h^a_{\mu\nu}$  $m_a^2 = \lambda_a$

vectors: A^i_μ  $m_i^2 = \lambda_i - \frac{2R_{(N)}}{N}$ Killing vectors massless (isometries)

scalars: ϕ^0 , ϕ^a $m_a^2 = \lambda_a - \frac{2R_{(N)}}{N}$ zero mode is volume modulus

scalars: $\phi^{\mathcal{I}}$ $m_{\mathcal{I}}^2 = \lambda_{\mathcal{I}} - \frac{2R_{(N)}}{N}$ zero modes massless (shape moduli)

Flat space spectrum

If we want to do S-matrix stuff, lower dimensional space should be flat

Internal manifold is Ricci flat: $R_{mn} = 0$

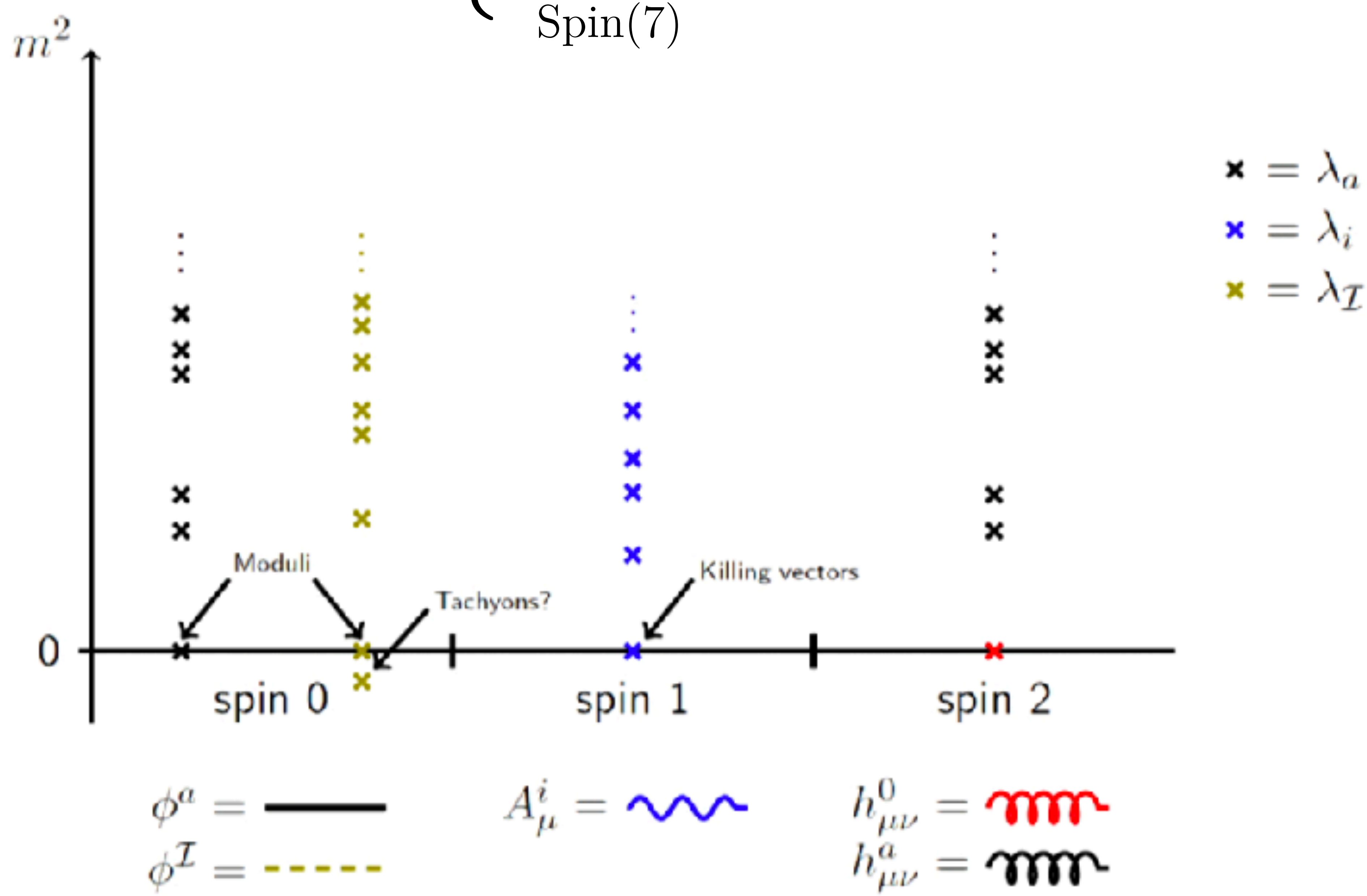
All have special holonomy.

Closed Ricci-flat manifolds are rare. The known examples are:

- flat tori
- Calabi-Yau's
- G_2
- Spin(7)

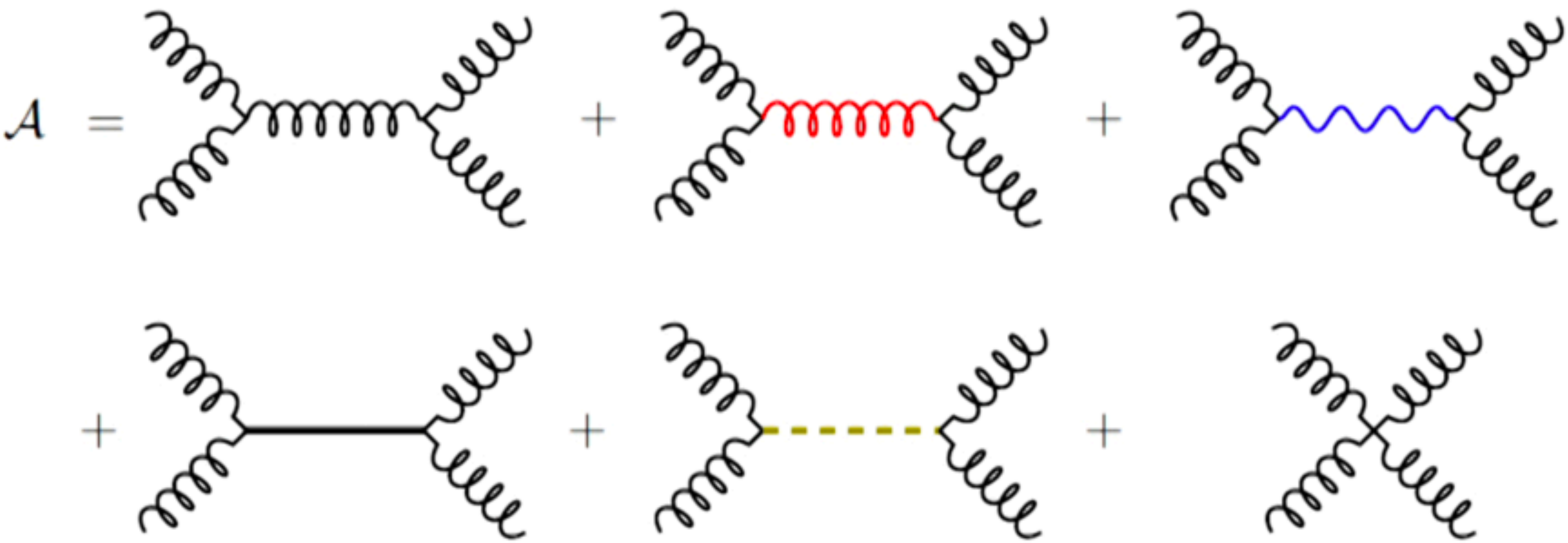
$\lambda_{\mathcal{I}} \geq 0$

Dai, Wang, Wei (2005)

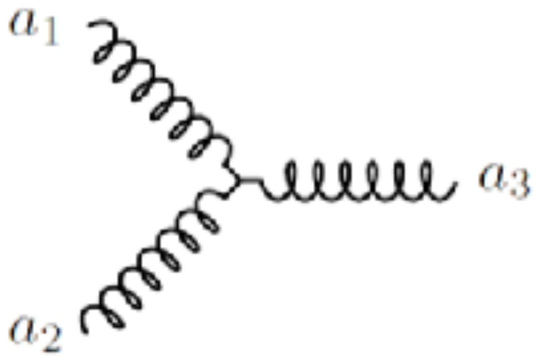


Massive spin-2 4-pt amplitude

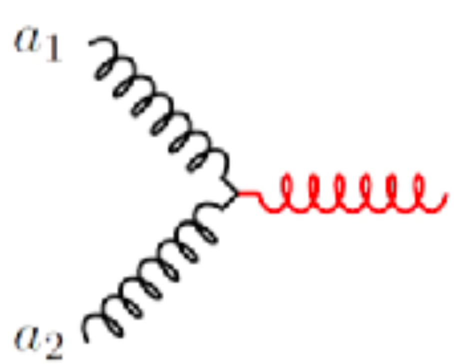
$$h^{a_1} h^{a_2} \to h^{a_3} h^{a_4}$$



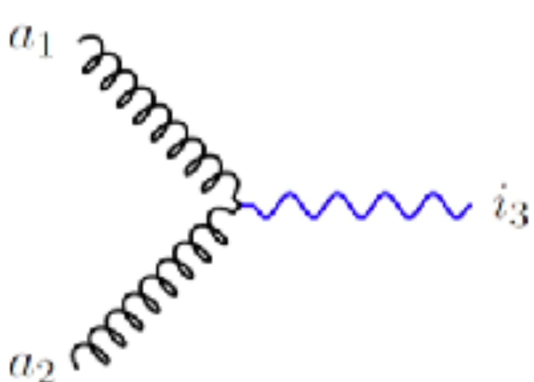
Cubic Interactions



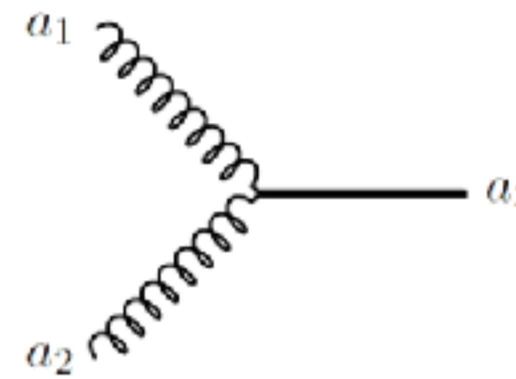
$$g_{a_1a_2a_3} \equiv \int_{\mathcal{N}} \psi_{a_1}\psi_{a_2}\psi_{a_3},$$



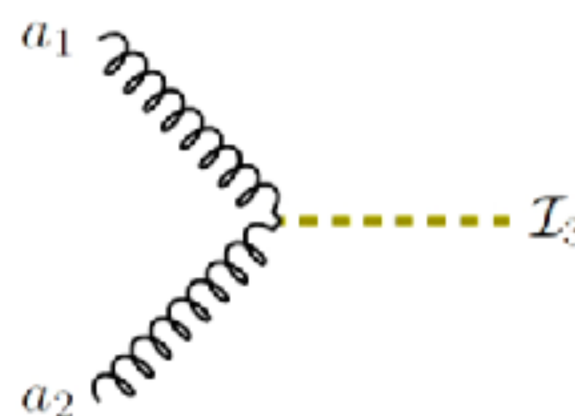
fixed minimal coupling $M_d^{d-2} = V M_D^{D-2}$



$$g_{a_1a_2i_3} \equiv \int_{\mathcal{N}} \partial^{n_1}\psi_{a_1}\psi_{a_2}Y_{n_1 i_3}$$

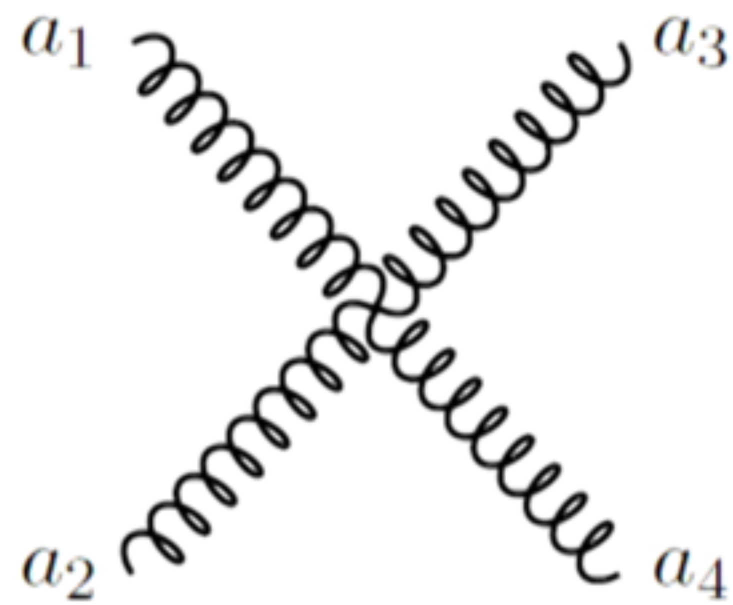


$$g_{a_1a_2a_3} \equiv \int_{\mathcal{N}} \psi_{a_1}\psi_{a_2}\psi_{a_3}$$



$$g_{a_1a_2\mathcal{I}_3} \equiv \int_{\mathcal{N}} \partial_n\psi_{a_1}\partial_m\psi_{a_2}h^{mn}_{TT,\mathcal{I}_3}$$

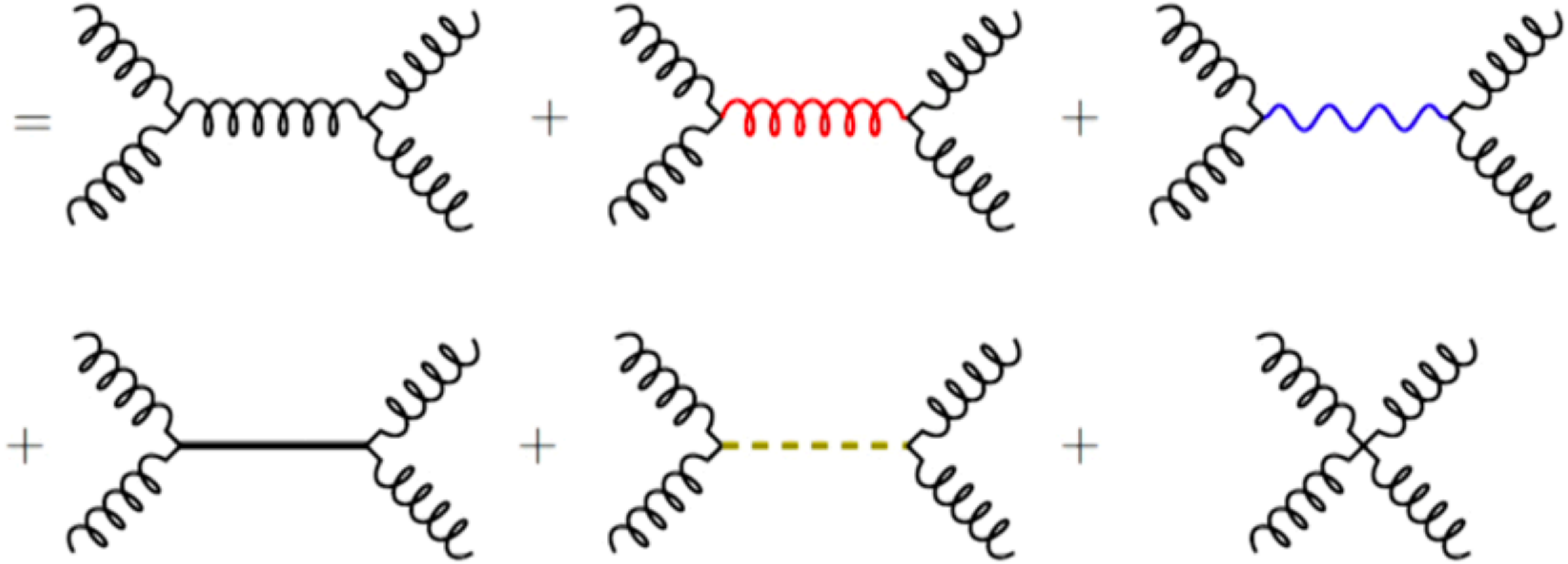
Quartic Interaction



$$g_{a_1 a_2 a_3 a_4} \equiv \int_{\mathcal{N}} \psi_{a_1} \psi_{a_2} \psi_{a_3} \psi_{a_4}$$

Full Amplitude

$\mathcal{A} =$



$$= \alpha_{10}E^{10} + \alpha_8 E^8 + \alpha_6 E^6 + \alpha_4 E^4 + \alpha_2 E^2 + \dots$$

$$-\frac{16(3+5\cos^2\theta)(d-2)^2}{(d-1)^2 M_D^{D-2}\lambda_{a_1}\lambda_{a_2}\lambda_{a_3}\lambda_{a_4}} E^{10}\sum_a g_{a_1 a_2}{}^{a} g_{a_3 a_4 a} + \dots$$

$\alpha_{10},\ \alpha_8,\ \alpha_6,\ \alpha_4$ must independently vanish

E^{10} sum rule

$\alpha_{10} = 0$ →

$$\begin{aligned} g_{a_1 a_2 a_3 a_4} &= \sum_a g_{a_1 a_2}{}^a g_{a_3 a_4 a} + V^{-1}\delta_{a_1 a_2}\delta_{a_3 a_4} \\ &= \sum_a g_{a_1 a_3}{}^a g_{a_2 a_4 a} + V^{-1}\delta_{a_1 a_3}\delta_{a_2 a_4} \\ &= \sum_a g_{a_1 a_4}{}^a g_{a_2 a_3 a} + V^{-1}\delta_{a_1 a_4}\delta_{a_2 a_3} \end{aligned}$$

Mathematical property of eigenfunctions that must hold on any Einstein manifold

Completeness: $$\overline{\psi_{a_1}\psi_{a_2}} = \sum_a g_{a_1 a_2}{}^a \psi_a + V^{-1}\delta_{a_1 a_2}.$$

Can use this to reduce any multi-overlap integral $\int_{\mathcal{N}} \psi_{a_1}\psi_{a_2}\cdots\psi_{a_k}$ to sums of triple overlaps

Multiple ways to do this → Associativity/crossing relations:

$$g_{a_1 a_2 a_3 a_4} = \int_{\mathcal{N}} \overline{\psi_{a_1}\psi_{a_2}}\,\overline{\psi_{a_3}\psi_{a_4}} = \int_{\mathcal{N}} \psi_{a_1}\psi_{a_2}\psi_{a_3}\psi_{a_4} = \int_{\mathcal{N}} \overline{\psi_{a_1}\overline{\psi_{a_2}\psi_{a_3}}\psi_{a_4}}$$

E^8 sum rule

$\alpha_8 = 0$ $\rightarrow$ $$\sum_a \left(4\lambda_{a_1} - 3\lambda_a\right) g^2_{a_1 a_1 a} + 4V^{-1}\lambda_{a_1} = 0,$$

(identical external flavors)

comes from crossing with 2 derivative insertions:

$$\int_{\mathcal{N}} \psi_{a_1}\psi_{a_1}\partial_m\psi_{a_1}\partial^m\psi_{a_1} = \int_{\mathcal{N}} \psi_{a_1}\psi_{a_1}\partial_m\psi_{a_1}\partial^m\psi_{a_1}$$

requires that a heavy tensor is exchanged, so there is an a^* such that

$$g_{a_1 a_1 a^*} \neq 0 \quad \text{and} \quad \frac{4}{3}\lambda_{a_1} < \lambda_{a^*} \implies \frac{2m_{\text{external}}}{\sqrt{3}} < m_{\text{exchanged}}.$$

repeat argument with internal particle now external → Unitarity requires an *infinite* tower of states

E^6 sum rule

$\alpha_6 = 0$ $\Rightarrow$

(identical external flavors)

$$\sum_a P_{E^6}\left(\lambda_a/\lambda_{a_1}\right)\lambda_{a_1}^2 g_{a_1a_1a}^2 + 16N(N-1)\sum_{\mathcal{I}} g_{a_1a_1\mathcal{I}}^2 = 0,$$

$$P_{E^6}(x) = (4-3N)Nx^2 + 4(N^2-3)x + 16.$$

comes from crossing with 4 derivatives:

$$\int_{\mathcal{N}} \partial_m\psi_{a_1}\partial_n\psi_{a_1}\partial^m\psi_{a_1}\partial^n\psi_{a_1} = \int_{\mathcal{N}} \partial_m\psi_{a_1}\partial_n\psi_{a_1}\partial^m\psi_{a_1}\partial^n\psi_{a_1}\,.$$

E^4 sum rule

$\alpha_4 = 0$ ⟹

(identical external flavors)

$$\sum_a P_{E^4}\left(\lambda_a/\lambda_{a_1}\right)\lambda_{a_1}^3 g_{a_1a_1a}^2 + 16(N-1)\sum_{\mathcal{I}} \lambda_{\mathcal{I}} g_{a_1a_1\mathcal{I}}^2 = 0,$$

$$P_{E^4}(x) = x(x-4)((3N-2)x - 4N).$$

comes from crossing with 6 derivatives:

$$\int_{\mathcal{N}} \partial_m \psi_{a_1} \partial_n \psi_{a_1} \Delta_L \left(\partial^m \psi_{a_1} \partial^n \psi_{a_1}\right) = \int_{\mathcal{N}} \partial_m \psi_{a_1} \partial_n \psi_{a_1} \Delta_L \left(\partial^m \psi_{a_1} \partial^n \psi_{a_1}\right)$$

E^4 sum rule

$$\sum_a P_{E^4}\left(\lambda_a/\lambda_{a_1}\right)\lambda_{a_1}^3 g_{a_1 a_1 a}^2 + 16(N-1)\sum_{\mathcal{I}} \lambda_{\mathcal{I}} g_{a_1 a_1 \mathcal{I}}^2 = 0,$$

$$P_{E^4}(x) = x(x-4)((3N-2)x-4N).$$

Assume $\lambda_{\mathcal{I}} \geq 0$.. Then first term must be ≤ 0, so there exists an eigenmode a^* such that

$$g_{a_1 a_1 a^*} \neq 0 \quad \text{and} \quad \frac{4N}{3N-2}\lambda_{a_1} \leq \lambda_{a^*} \leq 4\lambda_{a_1}.$$

→ For closed Ricci-flat manifolds with special holonomy,

$$\frac{\lambda_{k+1}}{\lambda_k} \leq 4,$$

where λ_k is the k^{th} nonzero eigenvalue of the scalar Laplacian.

Bounds the gaps between KK excitations of the graviton.
(No EFT with a finite number of massive gravitons from KK)

Also applies to smooth Calabi-Yau compactifications of string theory and G_2 compactifications of M-theory.

E^4 sum rule

Constraint on eigenvalue gaps for closed Ricci flat with $\lambda_{\mathcal{I}} \geq 0$.

$$\frac{\lambda_{k+1}}{\lambda_k} \leq 4$$

Includes all known cases of closed Ricci flat manifolds

This bound is optimal: it is saturated in every dimension by the first distinct nonzero eigenvalues on certain tori.

Example: Quintic Calabi-Yau (volume = 1) V. Braun, T. Brelidze, M. R. Douglas, and B. A. Ovrut (2008)

$$\lambda_k \in \{41.1 \pm 0.4, 78.1 \pm 0.5, 82.1 \pm 0.3, 94.5 \pm 1, 102 \pm 1\},$$

Einstein condition essential: general closed Riemannian manifolds have no such bound:

de Verdiere's theorem: given a closed manifold of dimension $N \geq 3$ and any finite sequence of non-decreasing positive numbers,

$$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_k,$$

then there exists a metric such that this is the sequence of the first k nonzero eigenvalues.

Geometry/CFT analogy

Einstein Manifolds		CFTs
eigenfunctions	ψ_a	primary operators
eigenvalues	λ_a	scaling dimensions
overlap integrals	$g_{a_1 a_2 \cdots a_k} \equiv \int_{\mathcal{N}} \psi_{a_1} \psi_{a_2} \cdots \psi_{a_k}$	correlators
covariant derivatives	∇_n	descendent operators
completeness	$\psi_{a_1}\psi_{a_2} = \sum g_{a_1 a_2}{}^{a} \psi_a + V^{-1} \delta_{a_1 a_2}$	OPE
sum rules	$\int_{\mathcal{N}} \psi_{a_1}\psi_{a_2}\psi_{a_3}\psi_{a_4} = \int_{\mathcal{N}} \psi_{a_1}\psi_{a_2}\psi_{a_3}\psi_{a_4}$	crossing relations
Lichnerowicz bound	$\lambda_a \geq \frac{R}{N-1},$	unitarity bound
Geometry data	$\lambda_a \quad , \quad g_{a_1 a_2 a_3}$	CFT data

Geometric bootstrap

Like CFT bootstrap, exploit crossing relations to constrain the data

Crossing relations for a general Einstein manifold (not necessarily Ricci flat):

$$\int_{\mathcal{M}} \partial_m \psi_{a_1} \partial^m \psi_{a_1} \psi_{a_1} \psi_{a_1} = \int_{\mathcal{M}} \partial_m \psi_{a_1} \partial^m \psi_{a_1} \psi_{a_1} \psi_{a_1}$$

$$\int_{\mathcal{M}} \partial_m \psi_{a_1} \partial_n \psi_{a_1} \partial^m \psi_{a_1} \partial^n \psi_{a_1} = \int_{\mathcal{M}} \partial_m \psi_{a_1} \partial_n \psi_{a_1} \partial^m \psi_{a_1} \partial^n \psi_{a_1},$$

$$\int_{\mathcal{M}} \partial_m \psi_{a_1} \partial_n \psi_{a_1} \Delta_L \left(\partial^m \psi_{a_1} \partial^n \psi_{a_1}\right) = \int_{\mathcal{M}} \partial_m \psi_{a_1} \partial_n \psi_{a_1} \Delta_L \left(\partial^m \psi_{a_1} \partial^n \psi_{a_1}\right)$$

$$V^{-1} \vec{F}_1 + \frac{1}{\lambda_{a_1}^2} \sum_{\mathcal{I}} \vec{F}_2\, g_{a_1 a_1 \mathcal{I}}^2 + \sum_{a \notin I_{\text{conf.}}} \left[\vec{F}_3 + \frac{R \vec{F}_4}{(N-1)\lambda_a - R}\right] g_{a_1 a_1 a}^2 = 0,$$

$$\vec{F}_1 = (4, -16, 0),$$

$$\vec{F}_2 = \left(0, 16N(N-1), 16N(N-1)\frac{\lambda_{\mathcal{I}}}{\lambda_{a_1}}\right),$$

$$\vec{F}_3 = \left(4 - \frac{3\lambda_a}{\lambda_{a_1}}, \frac{N\lambda_a}{\lambda_{a_1}}\left(4N + \frac{(4-3N)\lambda_a}{\lambda_{a_1}}\right), \frac{N\lambda_a}{\lambda_{a_1}}\left(4 - \frac{\lambda_a}{\lambda_{a_1}}\right)\left(4N - \frac{(3N-2)\lambda_a}{\lambda_{a_1}}\right)\right),$$

$$\vec{F}_4 = \left(0, \left(4 + \frac{(N-2)\lambda_a}{\lambda_{a_1}}\right)^2, \frac{\lambda_a}{\lambda_{a_1}}\left(4 + \frac{(N-2)\lambda_a}{\lambda_{a_1}}\right)^2\right).$$

Geometric bootstrap

postulate some candidate geometric data, (a collection of eigenvalues and triple overlap integrals)

↓

search for a constant vector $\vec{\alpha} \in \mathbb{R}^3$ such that the condition

$$V^{-1}\vec{\alpha}\cdot\vec{F}_1 + \frac{1}{\lambda_{a_1}^2}\sum_{\mathcal{I}}\vec{\alpha}\cdot\vec{F}_2\, g_{a_1 a_1 \mathcal{I}}^2 + \sum_{a\notin I_{\text{conf.}}}\left[\vec{\alpha}\cdot\vec{F}_3 + \frac{R\,\vec{\alpha}\cdot\vec{F}_4}{(N-1)\lambda_a - R}\right] g_{a_1 a_1 a}^2 = 0$$

can never be satisfied by this data.

↓

If such an $\vec{\alpha}$ exists, candidate geometric data is ruled out

problem of finding such an $\vec{\alpha}$ can be formulated as a semidefinite program (SDP)

D. Poland, D. Simmons-Duffin, A. Vichi (2011)

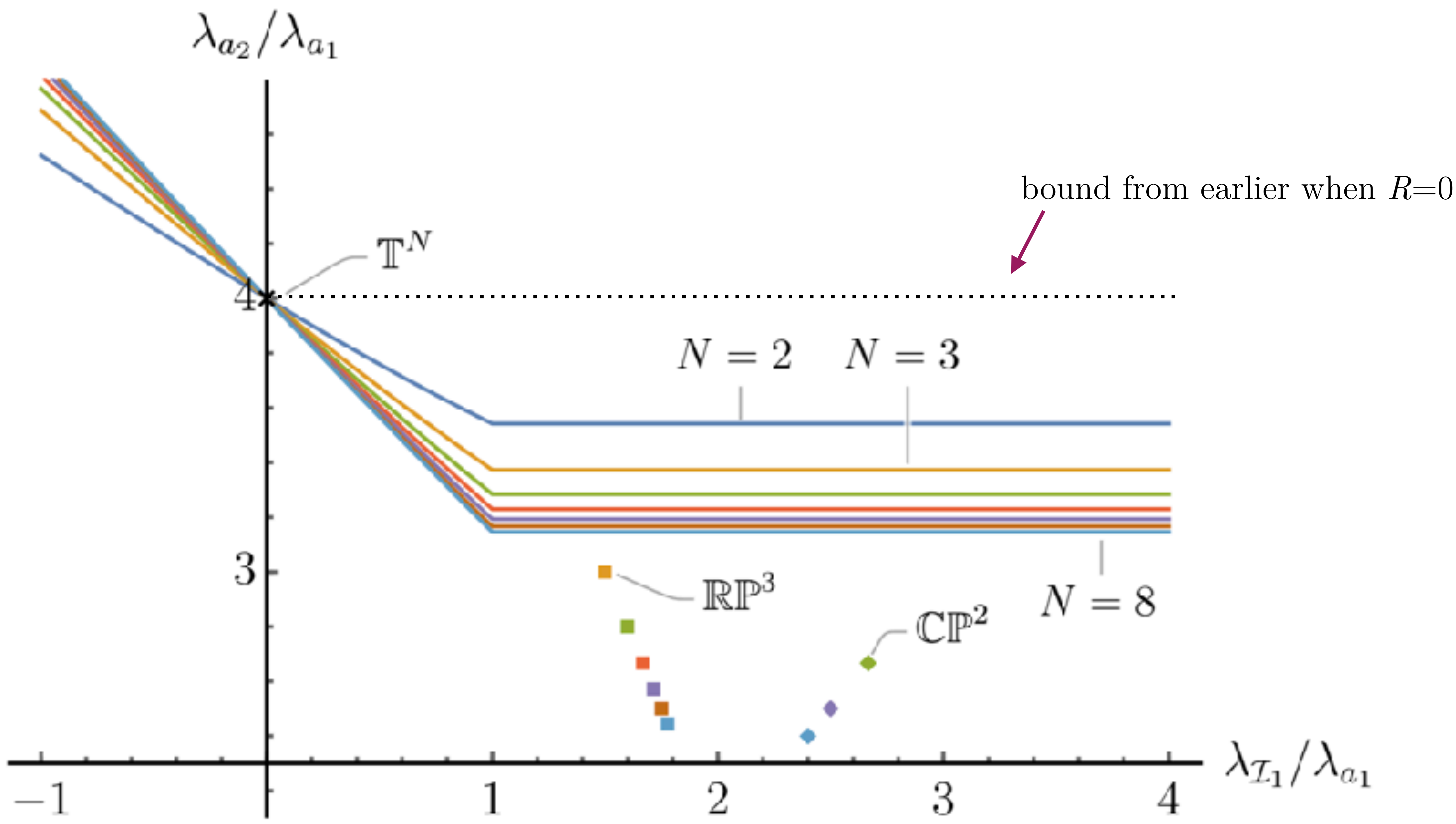
can be solved using SDPB, and Mathematica in simpler cases

D. Simmons-Duffin, (2015)

Bounds on eigenvalues

Upper bound on the ratio of the 2 lowest lying scalar eigenvalues, as a function of the smallest Lichnerowicz eigenvalue

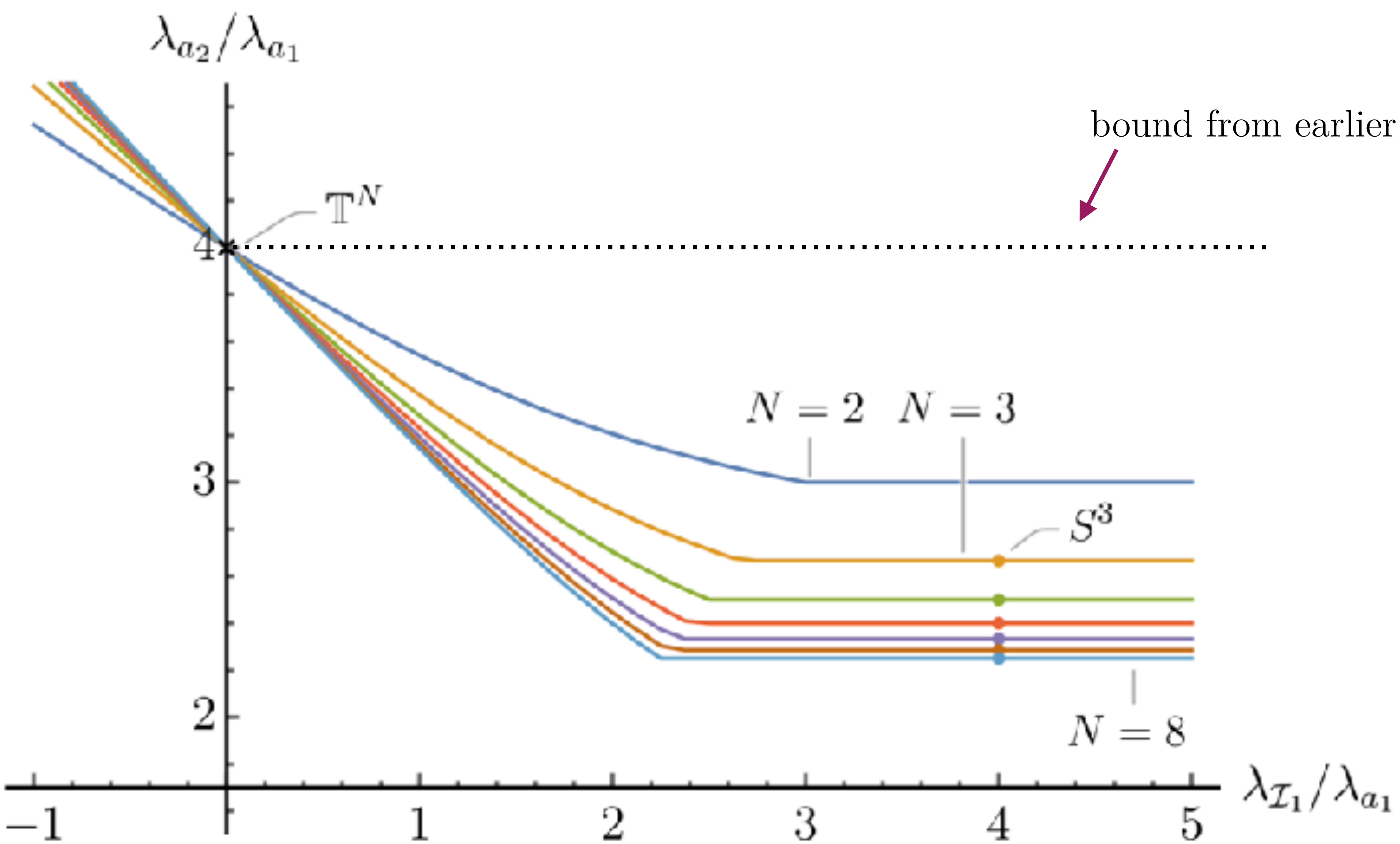
Assumptions: $R \geq 0$



Bounds on eigenvalues

Upper bound on the ratio of the 2 lowest lying scalar eigenvalues, as a function of the smallest Lichnerowicz eigenvalue

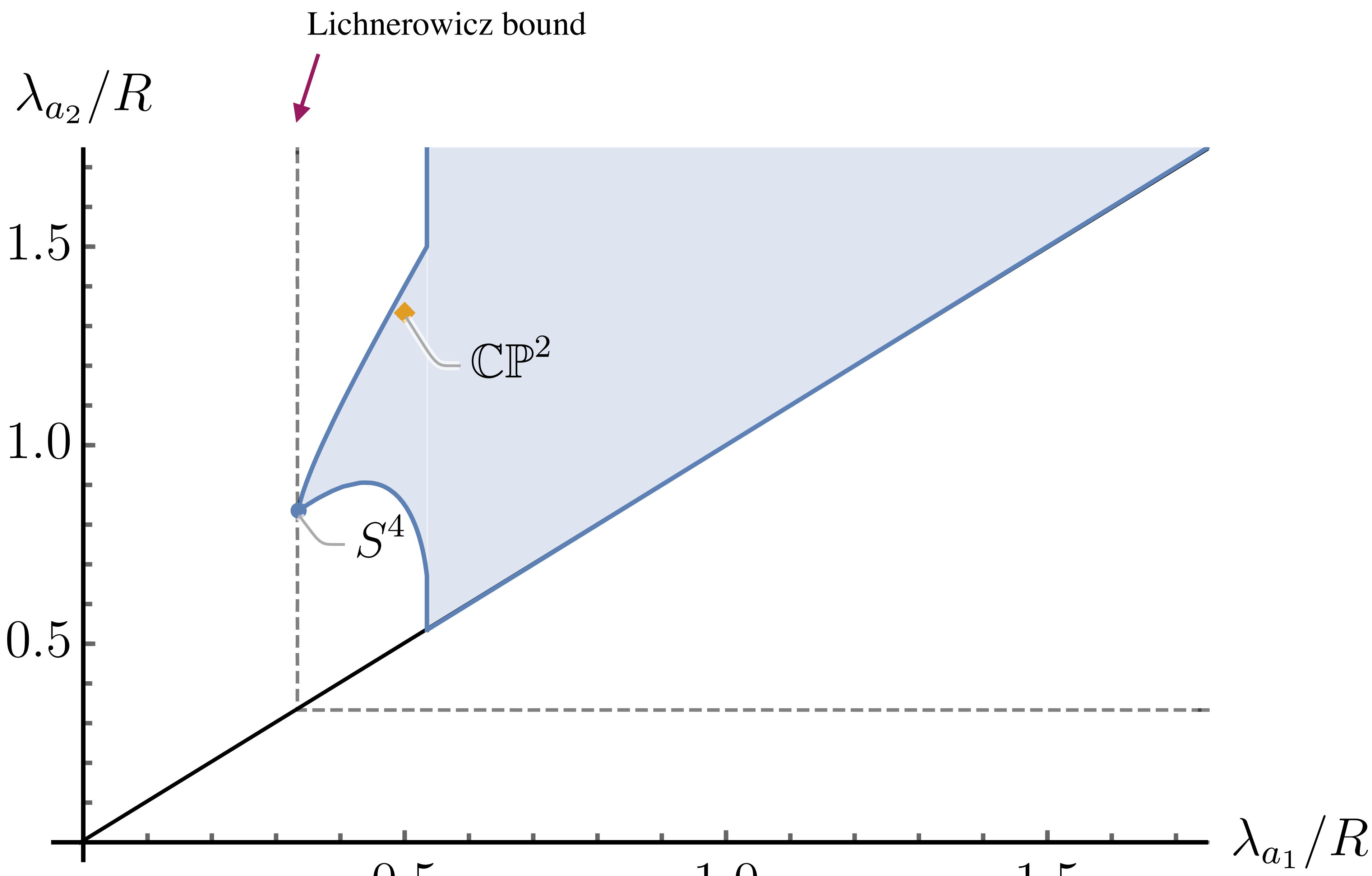
Assumptions: $R \geq 0$, $\mathbb{Z}_2$ symmetry under which ψ_{a_1} is odd



Bounds on eigenvalues

allowed values of the 2 lowest lying scalar eigenvalues, relative to the curvature

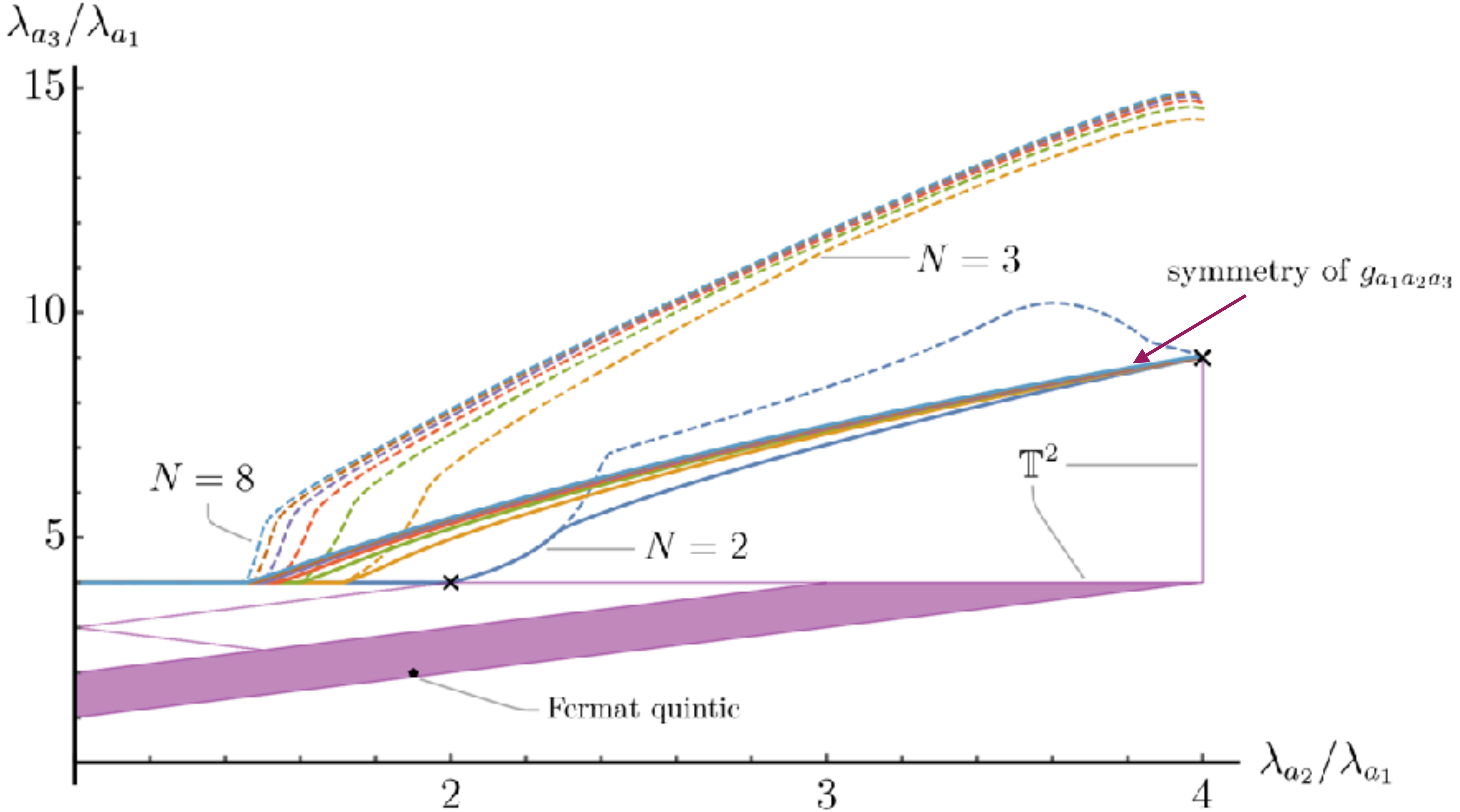
Assumptions: $N = 4$, $R > 0$, $\lambda_{a_3} \geq \frac{3R}{2}$, $\lambda_{\mathcal{I}} \geq \frac{4R}{3}$



Bounds on eigenvalues

Upper bound on the ratio of the 3rd to 1st scalar eigenvalue vs. the 2nd to 1st scalar eigenvalue

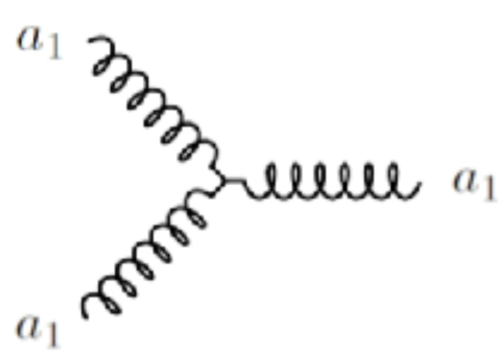
Assumptions: $R \geq 0\,,\ \lambda_{\mathcal{I}} \geq 0$



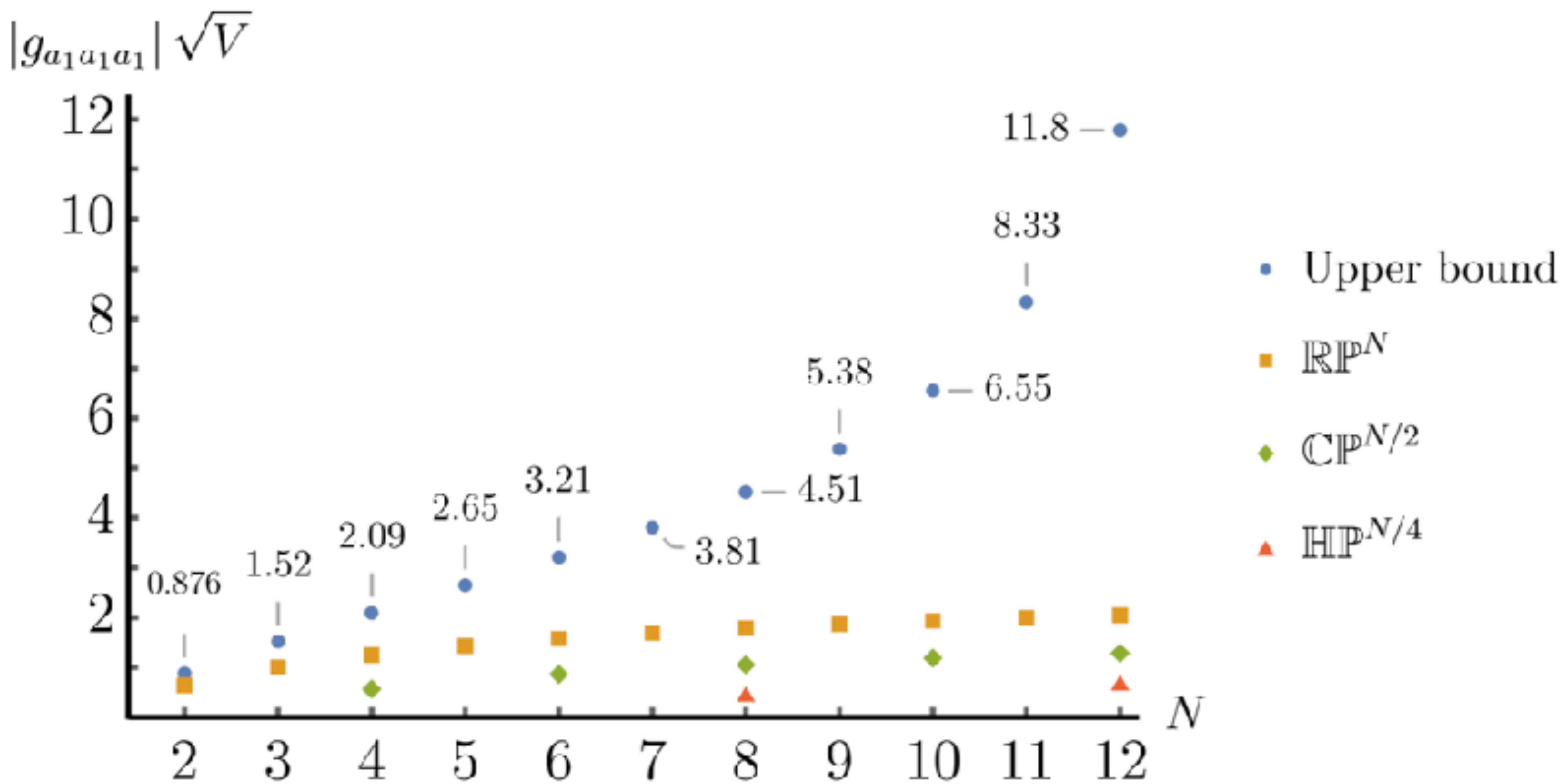
Bounds on cubic couplings

Upper bound on lightest
massive spin-2 self-coupling

Assumptions: $R \geq 0\,, \;\; \lambda_{\mathcal{I}} \geq 0$



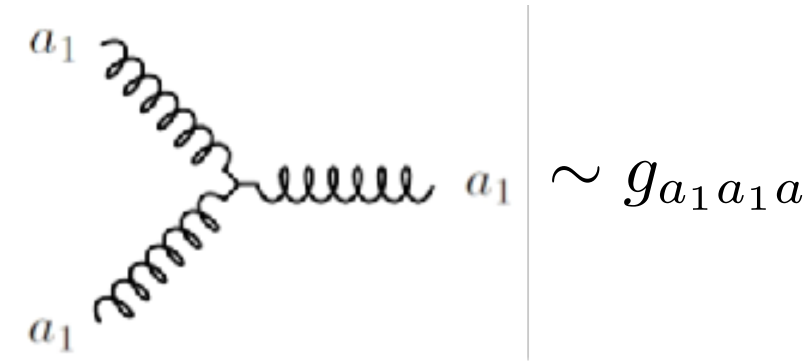
$\sim g_{a_1 a_1 a_1}$

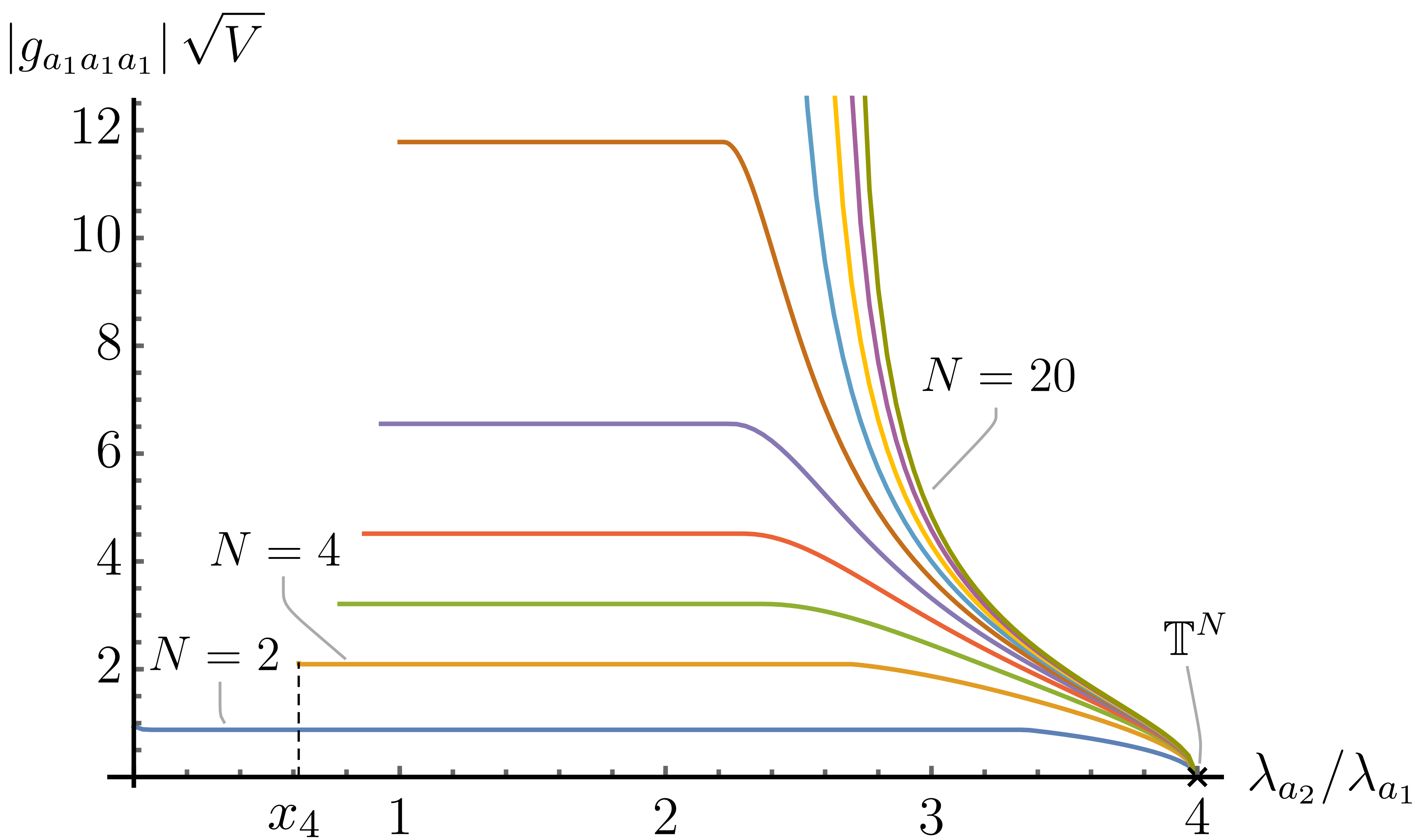


Bounds on cubic couplings

Upper bound on lightest massive spin-2 self-coupling as a function of the ratio of the lightest 2 eigenvalues

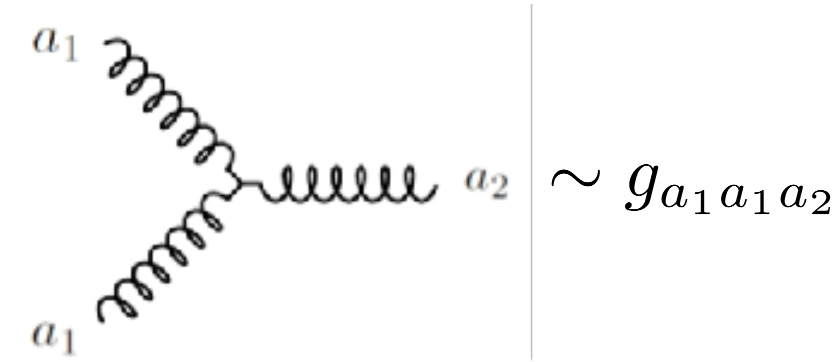
Assumptions: $R \geq 0$, $\lambda_{\mathcal{I}} \geq 0$



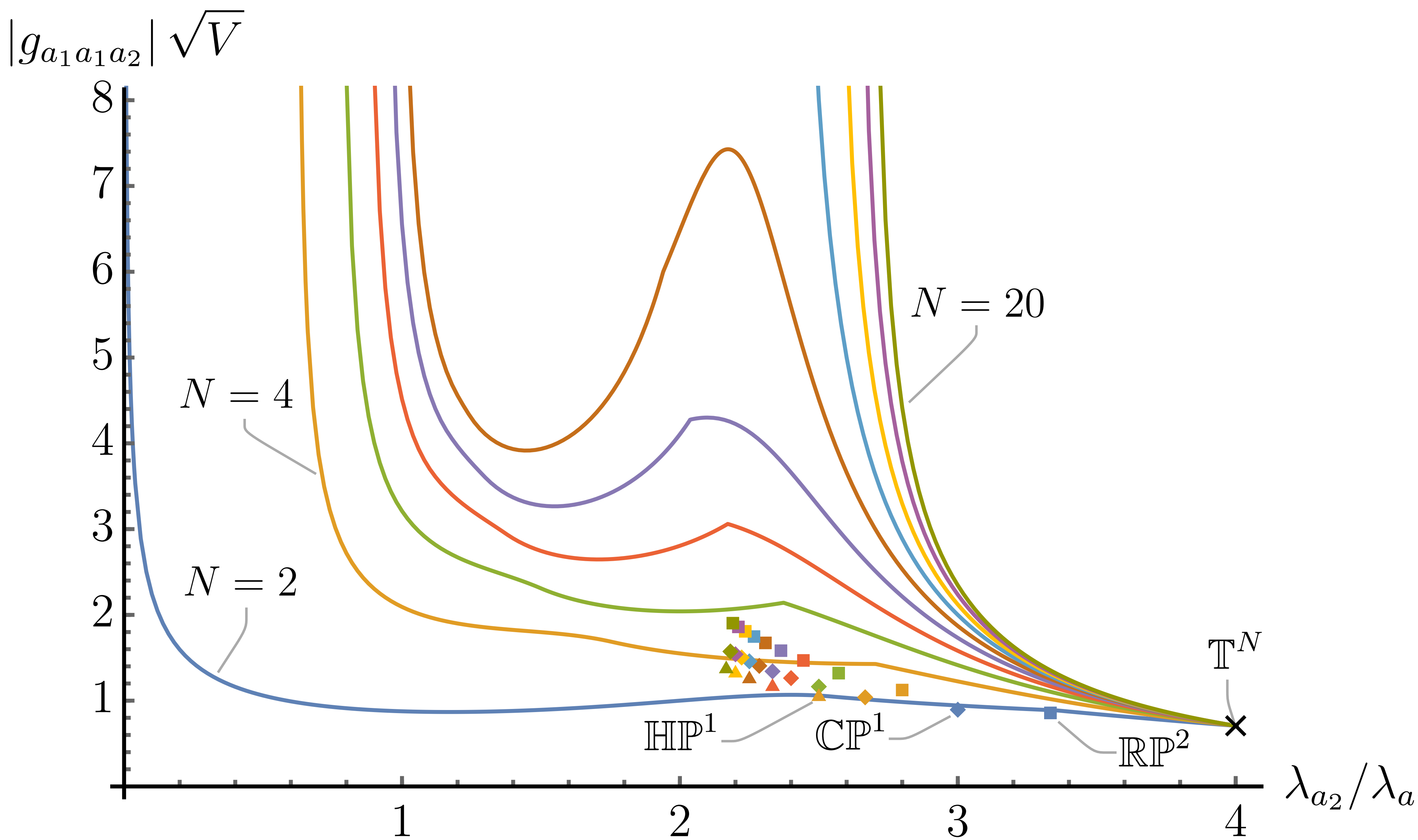


Bounds on cubic couplings

Upper bounds on massive spin-2 coupling
of lightest to next lightest mode



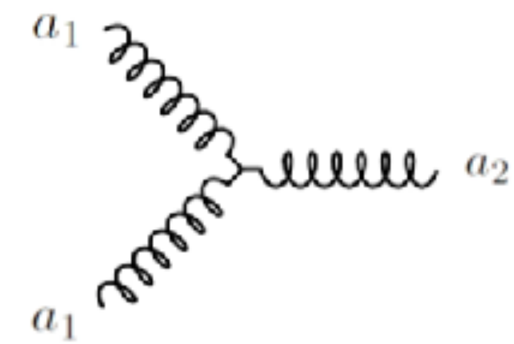
Assumptions: $R \geq 0\,,\ \lambda_{\mathcal{I}} \geq 0$



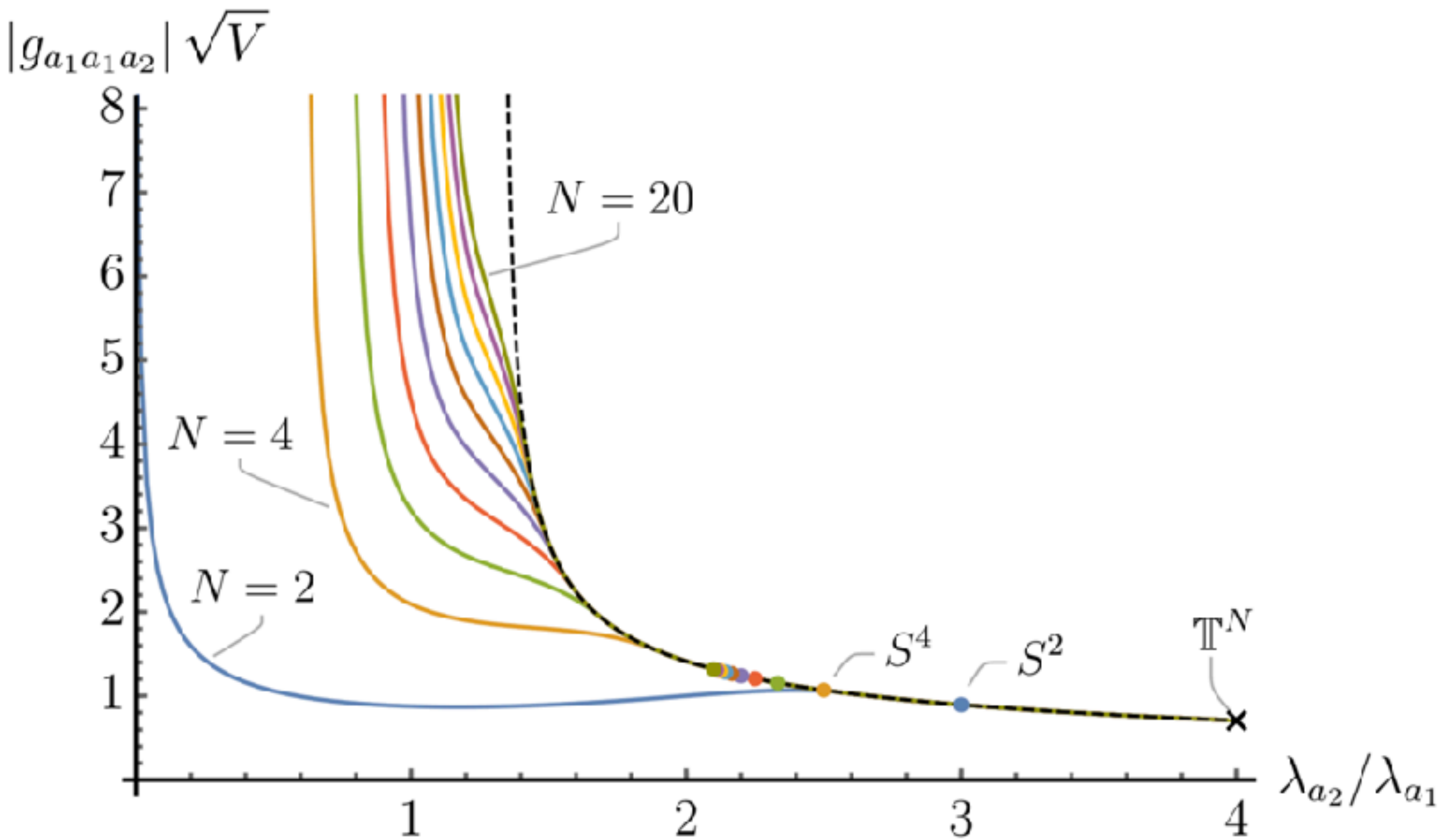
Bounds on cubic couplings

Upper bounds on massive spin-2 coupling of lightest to next lightest mode

Assumptions: $R \geq 0\,,\ \lambda_{\mathcal{I}} \geq 0\,,\ \mathbb{Z}_2$ symmetry



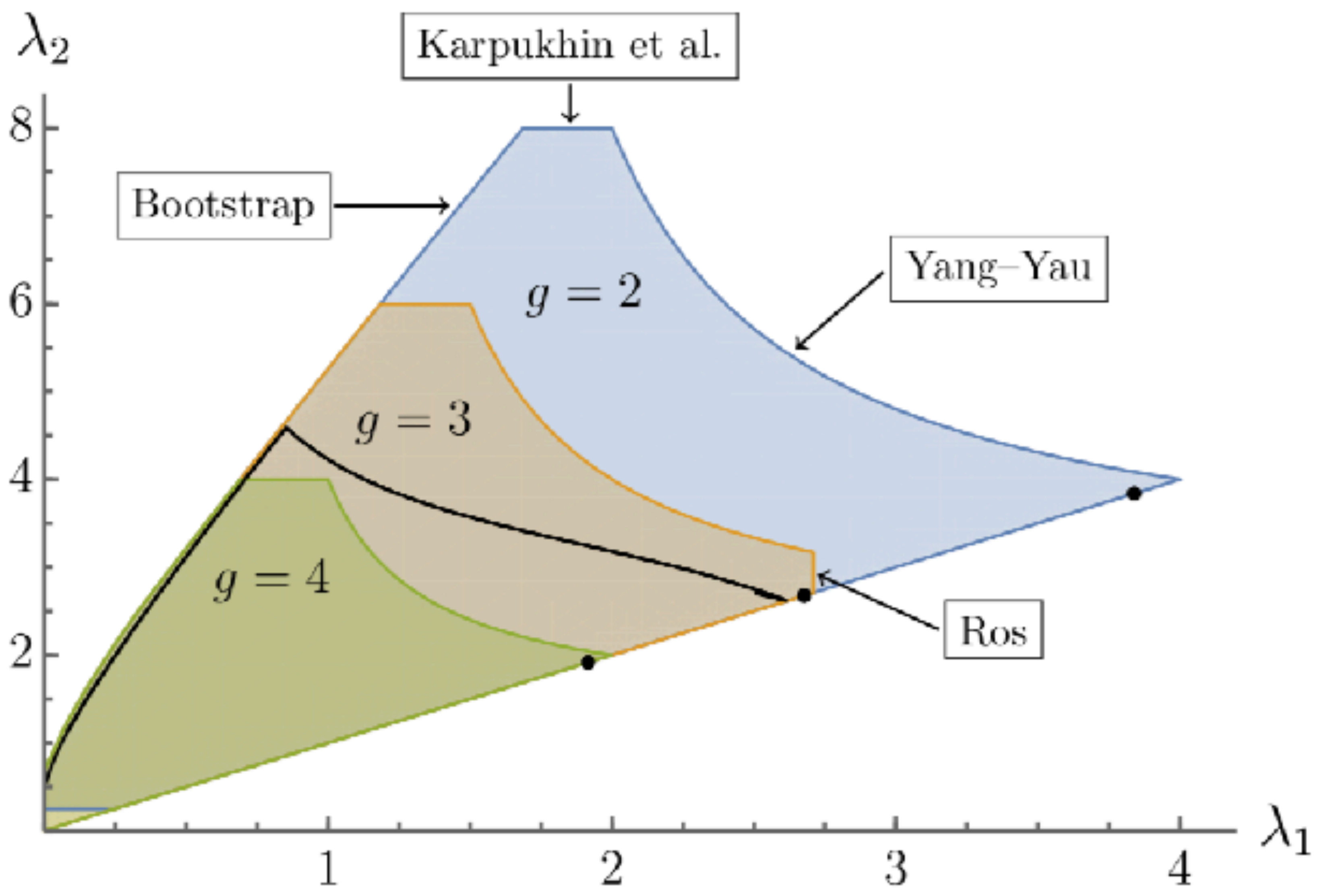
$\sim g_{a_1 a_1 a_2}$



Bounds on closed hyperbolic manifolds

James Bonifacio: arxiv:2107.09674

Allowed lowest eigenvalues on Genus g surfaces:



Conclusions and open questions

- New non-trivial constraints on the possible eigenvalue spectra and triple overlap integrals of Einstein manifolds

- They come from crossing relations on quadruple overlap integrals. The same relations ensure the correct high-energy behavior of KK reductions of gravity

- Do any non-trivial manifolds live at the kinks of our bootstrap bounds?

- Can any non-trivial manifolds be isolated by bootstrap bounds?

- Is a manifold uniquely determined by its geometric data? (can't hear the shape of a drum, but perhaps with triple overlaps?)

- Are there data that satisfy all the crossing relations which do not come from any manifold? (a non-geometric Kaluza-Klein compactification)